Contents

0.1 Definitions . . . 3
0.2 Theory . . . 5
0.3 Chebyshev Polynomials of the first kind . . . 7
0.4 Legendre Polynomials . . . 8
0.5 A priori estimates of the Schauder Type . . . 10
0.6 Notations . . . 11

1 Inverse Problems 13
1.1 Inverse problem of gravimetry . . . 13
1.2 Inverse scattering . . . 14
1.3 The inverse conductivity problem . . . 15
1.4 Tomography (Integral geometry) . . . 16
1.5 Inverse spectral problems . . . 17

2 Inverse parabolic problems 19
2.1 Final overdetermination . . . 21
2.2 Lateral overdetermination:single measurements . . . 27
2.3 Lateral overdetermination:many measurements . . . 28
2.4 Interior sources . . . 31

3 Solution Strategies 35
3.1 Numerical solution for the inverse heat problem in R . . . 35
3.2 Solving the one dimensional inverse parabolic problem using Chebyshev polynomials of the first kind . . . 37
3.3 Numerical solution of an inverse diffusion problem based on the Laplace transform and the finite difference method . . . 41
3.4 A tau method for solving the one-dimensional parabolic inverse problem based on the shifted Legendre polynomials . . . 43

3.5 A high-order compact finite difference method for solving an inverse problem of the one-dimensional parabolic equation . . 45

4 Inverse parabolic problems with discontinuous principal coefficient 51
4.1 Discontinuous principal coefficient 51
4.2 Regularization 68
4.3 Numerical solution for an inverse diffusion problem 69
4.4 Regularization by the Laplacian operator 76

5 Solving inverse parabolic problems using Adomian decomposition method 78
5.1 Adomian decomposition method 78
5.1.1 A general description of the ADM 79
5.1.2 Applications 80
5.2 Solution of some parabolic inverse problems by ADM 82
5.2.1 Parabolic inverse problems with unknown boundary conditions 82
5.2.2 Inverse parabolic problem with unknown control function 85
References 87